# COMMENTS

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## Comment I on "Instability threshold in the Bénard-Marangoni problem"

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The mathematical content of the linear stability study [L. E. Rabin, Phys. Rev. E **53**, R2057 (1996)] duplicates the early work of Pearson [J. Fluid Mech. **4**, 489 (1958)] and Nield [J. Fluid Mech. **19**, 34 (1964)]. It is shown that the physical conclusions reflect only a redefinition of the Marangoni number.
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The author of Ref. [1] claims to have found a new "correct" threshold (Ma=222.54, $k=2.33$) for the onset of surface-tension-driven Bénard convection as opposed to the classical "wrong" result (Ma=79.59, $k=1.99$) [2,3]. This claim cannot be justified, since the mathematical model treated in Ref. [1] is identical to the models used in Refs. [2,3] up to a redefinition of the dimensionless parameters. We demonstrate this below by showing that Ref. [1] predicts the same value for the relevant physical parameter $\Delta T=T_0-T_s$ (the difference between the temperature $T_s$ at the free surface and the temperature $T_0$ at the bottom) as do the classical works [2,3].

In Ref. [1] the Marangoni number is defined as

$$\mathrm{Ma}=\frac{\alpha(T_0-T_1)h}{\rho\nu\kappa} \tag{1}$$

(where $T_1$ denotes the temperature of the ambient gas). With the help of Eq. (2) from Ref. [1] we can write the above defined temperature difference in the form

$$\Delta T=(T_0-T_1)\,\frac{rh}{\kappa+rh}\,. \tag{2}$$

Inserting this into the expression for the neutral stability curve [Eq. (10) of Ref. [1]] and returning to dimensional quantities we obtain

$$\Delta T=\left(\frac{\rho\nu\kappa}{\alpha h}\right)\frac{4k[(rh/\kappa)\sinh k+k\,\cosh k](\sinh 2k-2k)}{(\sinh^3 k-k^3\cosh k)} \tag{3}$$

as Rabin's prediction for the critical temperature difference.

In Ref. [2] the Marangoni number is defined as

$$\mathrm{Ma}=\frac{\alpha\beta h^2}{\rho\nu\kappa} \tag{4}$$

[where $\beta=(T_0-T_s)/h$ is the basic temperature gradient]. The temperature difference is

$$\Delta T=\beta h. \tag{5}$$

Inserting this expression into Pearson's neutral stability curve [Eq. (27) in Ref. [2]] we are led to

$$\Delta T=\left(\frac{\rho\nu\kappa}{\alpha h}\right)\frac{4k[(rh/\kappa)\sinh k+k\cosh k](\sinh 2k-2k)}{(\sinh^3 k-k^3\cosh k)} \tag{6}$$

as Pearson's prediction for the critical temperature difference, identical to Eq. (3). All numerical results of Ref. [1], in particular, Figs. 1 and 2, can be obtained from the data of Refs. [2,3] by returning to the original definitions of Marangoni and Rayleigh number.

The correctness of Pearson's theory is supported by accurate experiments [4,5] and by direct numerical simulation [6,7].

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